



Ultrasonic velocity profiler applied to explore viscosity–pressure fields and their coupling in inelastic shear-thinning vortex streets

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Abstract

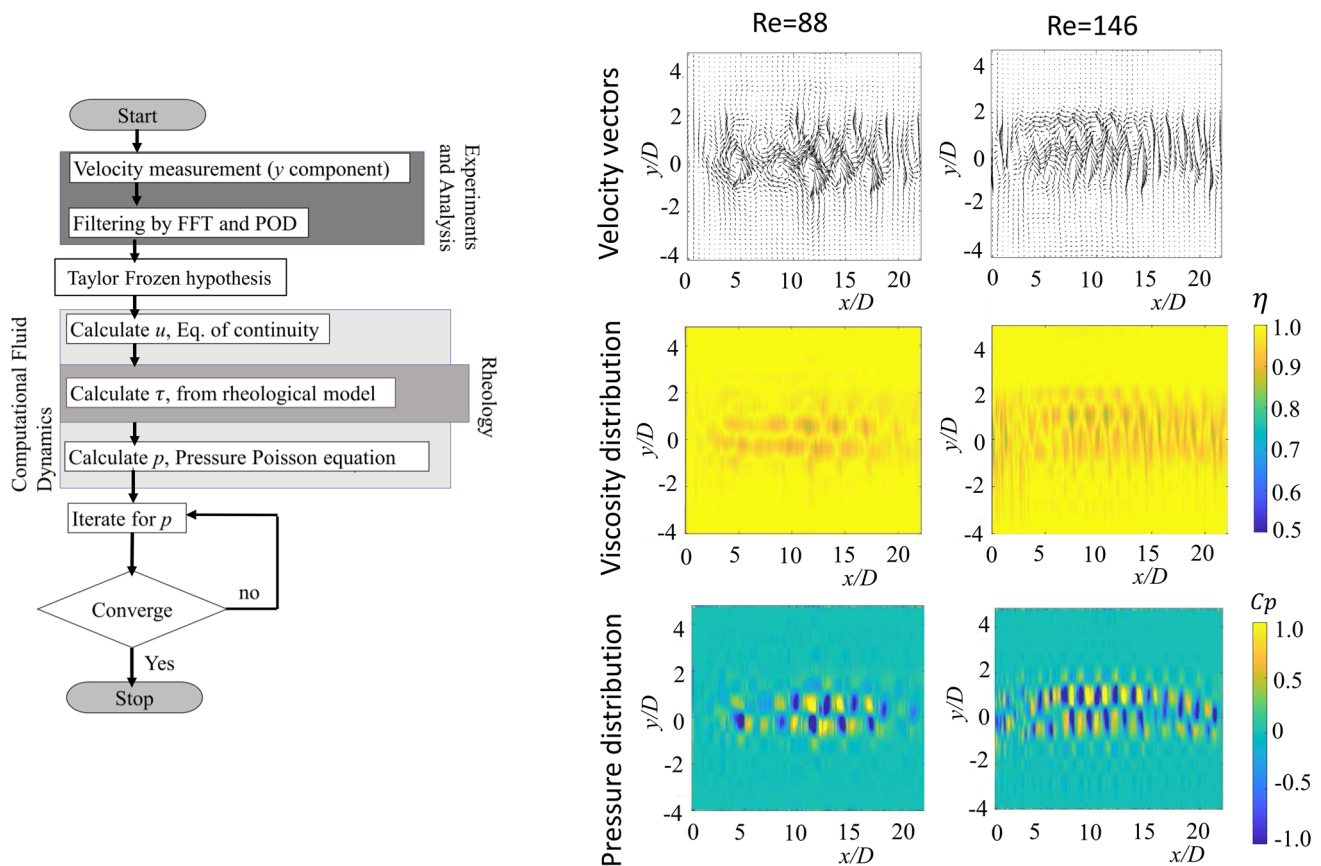
A method for simultaneous estimation of viscosity and pressure fields in inelastic shear-thinning fluids is developed by means of ultrasound velocity profiling technique (UVP). In the method, equation of continuity, rheological model and pressure Poisson equation are incorporated as data processing sequences for measured velocity distributions. The proposed method is applied to study the vortex street structure formed behind a circular cylinder, which shows viscosity–pressure coupling due to shear-thinning property of fluid. For demonstration, aqueous solution of CMC (carboxy methyl cellulose) of weight concentration of 0.1% is chosen as the working fluid with shear-thinning property. An alternating staggered pattern of low-pressure spots is successfully reconstructed for zero-shear-based Reynolds number, $Re = 50\text{--}300$. We have found that increasing Re resulted in decrease in vortex shedding Strouhal number because of vortex sustainability supported by shear-thinning property.

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Graphical abstract



1 Introduction

Flow over cylinder is a widely studied topic for Newtonian fluid flows (Williamson 1996) due to its relevance in many engineering problems. Large amount of research exists investigating physics of phenomena and flow control (Zdravkovich 1997) by theoretical (Mittal et al. 2008), experimental (Norberg 1994) and numerical simulations (Rajani et al. 2009). Comparatively, fewer studies have been conducted for non-Newtonian fluids. Main concern in working with non-Newtonian fluids is that they do not follow Newton's law of viscosity and exhibit various relations between stress and strain rate. Accordingly, these fluids are classified as shear thinning, shear thickening, viscoelastic, thixotropic and rheopectic. Purely, shear-thinning or thickening fluids can be represented by a generalized Newtonian fluid model where local shear stress is a function of local shear rate, and viscosity can vary spatially in flow. To simulate flow over a cylinder in shear-thinning fluids, such models like, power law, Casson, Cross and Carreau–Yasuda

model are generally used. For power law fluids, cylinder flow simulations were parametrically investigated in steady confined (Rao et al. 2011), unconfined (Sivakumar et al. 2007), circular (Bharti et al. 2007b, a; Bharti et al. 2008), square (Dhiman et al. 2006, 2007) and elliptical geometries (Koteswara Rao et al. 2010). However, power law model overpredicts viscosity for flows where shear rate is very low (e.g., free stream regions in external flow) and underpredicts viscosity when shear rate is very high. Carreau–Yasuda and Cross models incorporate zero and infinite shear rate viscosity and thus are more appropriate to represent viscosity over a wide range of shear rate (Carreau 1972) (Cross 1968). For Carreau fluids, (Pantokratoras 2016) reported drag coefficient variations with model parameters. These studies were mostly limited to steady flows. Recently, (Bailoor et al. 2019) have performed numerical simulation for unsteady cylinder flow by immersed boundary method. Force coefficients and vortex characteristics with Carreau number in shear-thinning fluid were investigated at $Re = 100$. More recently, flow over impulsively moving cylinder has also been simulated using Carreau model (Yun et al. 2020) where

the effect of Carreau number, power law index and Re on unsteady non-Newtonian fluid flow has been studied. There is a recent trend of simulations with models other than power law. On the other hand, experiments were performed for flow over cylinder of various shear-thinning fluids by Coelho and Pinho (2003). They have measured velocity using laser Doppler velocimeter (LDV) which provides point measurement of fluid velocity and reported point pressure measurements (Coelho and Pinho 2004). The details of unsteady wake flow field, viscosity and pressure fields have not been investigated experimentally for shear-thinning fluids which are the motivation for this work.

Ultrasound velocity profiling (UVP) and particle image velocimetry (PIV) can provide the velocity information in 1D and 2D/3D, respectively, but their application to wake flows of shear-thinning fluid flows is scarce. Note that PIV and UVP cannot directly provide the pressure field information which is important in investigation of interaction of rheological property with dynamic pressure via velocity gradient tensor field. Some of the available pressure measurement devices such as piezoelectric transducer, manometer and pitot tube can provide pressure information only at a single point in fluid. These devices are applicable to non-Newtonian fluid flows but are intrusive in nature. Pressure sensitive paint (PSP) (Bell et al. 2001) is non-intrusive and can be utilized to obtain pressure distribution over a solid surface but is not applicable in fluid flow domain. Table 1 classifies previous studies applied to cylinder wake flows. The studies described in Table 1 are purely experimental or numerical, except our previous paper (Tiwari et al. 2021) and (Tiwari et al. 2019). Tiwari et al. 2021 discussed PIV-based pressure and viscosity investigation in pseudoplastic flow for steady flows and focused on enlightening the change of vortex dynamics due to shear-thinning property of fluid. A linear error propagation analysis was also discussed to compare the characteristics of error in a shear-thinning fluid and a Newtonian fluid. In our previous work (Tiwari et al. 2019), UVP was applied to study flow over

cylinder in opaque Newtonian fluid at $Re = 1000$. We have developed a pressure estimation algorithm based on UVP measurements targeting vortex streets formed behind a cylinder. Equation of continuity and Navier–Stokes equations were incorporated into data processing of measured 1D-1C velocity distribution to reconstruct vortex street structure as well as corresponding pressure distribution.

The theory behind PIV-based pressure estimation is discussed below. From measured velocity field, pressure field can be estimated by substituting it into governing equations of fluid flows. For Newtonian fluids, PIV-based pressure field estimation methodology has been well established (van Oudheusden 2013). Its principle relies on substitution of measured velocity data into one of the equations describing the velocity–pressure relation is given below.

Navier–Stokes equation: Relation between pressure and velocity is prescribed through the Navier–Stokes equation which can give local instantaneous pressure gradient vector as

$$\nabla p = -\rho \left\{ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right\} + \mu \nabla^2 \mathbf{u} \quad (1)$$

where $\mathbf{u}$ and μ are fluid flow velocity vector and viscosity, respectively. Right hand side terms of the equation are provided by velocity measurements from PIV. By spatially integrating the local pressure gradient, scalar pressure field can be reconstructed. In literature, either Eulerian (Van Der Kindere et al. 2019) or Lagrangian (Charonko et al. 2010) methods are adopted depending on the sampling rate of velocity measurements. In our last work (Tiwari et al. 2021), we used a momentum equation with Eulerian method to reconstruct the pressure field albeit for a non-Newtonian fluid with variable viscosity.

Pressure Poisson equation: For incompressible flows, Poisson equation for pressure is derived by taking divergence of Eq. (1) and is written as

Table 1 Classification of research in literature on cylinder flow in shear-thinning fluid

Method	Experiment			Numerical			
Type	LDV	PIV	UVP	RANS	LES	DNS	IBM
Published studies	Coelho and Pinho (2003)	Tiwari et al. (2021)	Nakashima et al. (2016) Tiwari et al. (2019)	Bharti et al. (2008) Pantokratoras (2016)	-	Lashgari et al. (2012)	Bailoor et al. (2019)
Data dimension	0D(Point)	2D/3D	1D	2D/3D	2D/3D	2D/3D	
Nature	Real			Modelling			
Application to opaque fluid	No	No	Yes	Yes			
Steady/unsteady	Unsteady	Steady	–	Steady	–	Steady	Unsteady

LDV: laser Doppler velocimetry, PIV: particle image velocimetry, UVP: ultrasound velocity profiler, RANS: Reynolds averaged Navier–Stokes equations, LES: large Eddy simulations, DNS: direct numerical simulations, IBM: immersed boundary method

$$\nabla^2 p = -\rho \nabla \cdot (\mathbf{u} \cdot \nabla) \mathbf{u} \rightarrow -2\rho \left(\frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right) \text{ if } 2D \quad (2)$$

This equation does not contain the time derivative term. Viscous term in Eq. (1) also vanishes in Eq. (2) for Newtonian fluids. This avoids noise amplification due to second-order derivatives of PIV data of low velocity resolution.

For non-Newtonian fluids, these equations cannot be applied directly. In industry relevant opaque fluids, PIV is also not applicable. Thus, ultrasound velocity profiler (UVP) (Takeda 2012; Shiratori et al. 2015) is a promising and valid choice. UVP uses a single ultrasound transducer to provide one-dimensional, one-component (1D-1C) velocity distributions. Therefore, majority of UVP measurement targets one-directional dominant flow fields such as pipe flows or channel flows. Our team has also developed a novel rheometry using UVP (Yoshida et al. 2017, 2019). It is based on substitution of UVP-measured velocity distribution into rheological model to estimate complex rheological properties. Various physical relations between input and output quantities in flow structure analysis are illustrated in Fig. 1 for easy understanding of the present concept. The cases (a) to (d) of the figure had been established in literature as discussed above.

In this study, we propose a novel algorithm for UVP to simultaneously estimate viscosity and pressure fields in vortex streets formed behind cylinder for unsteady flow of inelastic shear-thinning fluids which may also be opaque. We present how UVP data can be best processed for evaluating viscosity–pressure correlations. It is emphasized that our main objective is not to reveal rheological property of unknown fluid, but is to establish a pressure field estimation method that couples with non-Newtonian fluid properties, using a known rheological constitutive equation that approximates the targeted fluid properties.

The outline of paper is as follows: In Sect. 2, the experimental arrangement and measurement systems are discussed. The algorithm is explained along with numerical governing equations and rheological model in Sect. 3. In Sect. 4, the results are discussed, and correlation plots are analyzed to understand the dependence of viscosity on enstrophy and pressure. Present technique is further tested at various Re in the range $50 < Re < 300$.

2 Experimental setup

A schematic of the experimental arrangement for velocity measurement in wake of a cylinder is shown in Fig. 2. A towing tank of dimensions 5.00 m in length, 0.54 m in width and 0.45 m in depth (h) was used. The diameter D and the length of cylinder are 0.04 m and 0.40 m, respectively. The

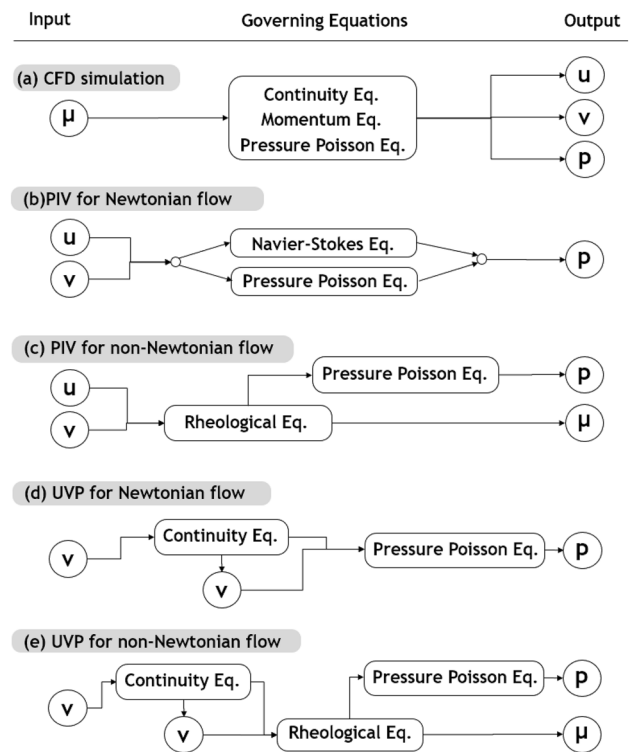


Fig. 1 Relations between input and output quantities depend on measurement information. u and v denote streamwise and perpendicular velocity components in two-dimensional flow, respectively. p and μ are pressure and viscosity distributions. Case **a** is a standard CFD simulation for Newtonian fluids, where fluid property is input to compute velocity and pressure. **b** PIV-based pressure field estimation for Newtonian flows. **c** for non-Newtonian flow (Tiwari et al. 2021). **d** UVP for Newtonian flow (Tiwari et al. 2019). **e** UVP for non-Newtonian flow which is the present topic

cylinder is made of acrylic resin and fixed on a mobile carriage with vertical shafts. The position of the cylinder is set horizontally at $h/2$ ($> 5D$) from free surface of fluid as well as the bottom floor. The effect of free surface is negligible if the cylinder is placed 3–4 diameters below the free surface (Abdul Khader 1979). The influence of bottom wall is also minimal because the effective blockage ratio is more than 10 (Bharti et al. 2007a). Hence, effect of free surface or bottom plane on the wake of cylinder is also neglected in this study.

2.1 Test fluid and experimental conditions

The test fluid is an aqueous CMC (carboxy methyl cellulose) solution of 0.1% by weight. Structural formula of CMC is shown in Fig. 3, in which R depends on degree of etherification (DoE). Manufacturer of CMC is Daicel FineChem Ltd, Japan.

Table 2 shows nominal viscosity of the CMC solutions for 19 types of product codes, measured by the manufacturer using a cone-type rheometer at 25 °C in temperature.

Fig. 2 A schematic of experimental setup with UVP measurement system. Ultrasound transducer is set vertically to measure y component of velocity in the wake of a circular cylinder, which is carried together with the cylinder along a linear slider attached on the container

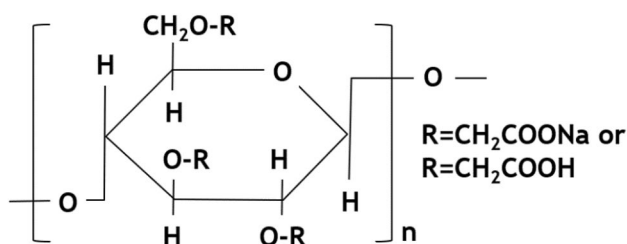
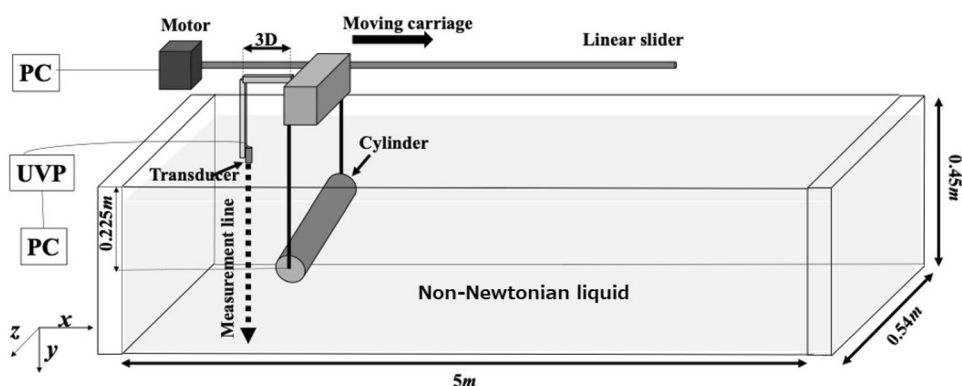


Fig. 3 Structural formula of carboxy methyl cellulose

Table 2 Viscosity of aqueous CMC solution manufactured by Daicel Co. Ltd

Degree of etherification	Code	Viscosity [mPaS] at 25 °C		
		1.0 wt. %	0.5 wt. %	0.1 wt. %
0.7	1110	20	3	1.1
	1120	35	5	1.1
	1130	75	12	1.2
	1140	150	25	1.4
	1150	250	42	1.7
	1160	400	68	2.1
	1170	650	110	2.8
	1180	1150	195	4.2
	1190	1650	280	5.6
0.9	1220	15	2	1.0
	1240	35	5	1.1
	1250	55	9	1.2
	1260	105	17	1.3
1.2	1330	75	12	1.2
	1350	250	42	1.7
	1380	1500	255	5.2
	1390	3500	595	10.8
1.0	2220	2250	382	7.3
	2260	5000	850	15.0

*Viscosity is representative value in each classified range measured by E-type rheometer

We chose the CMC of the code 1390 (indicated in bold) in Table 2, that has molecular weight of 2×10^6 kg/kmol, because our group also collected rheological data for this type (Yoshida et al. 2019) and confirmed that the measured viscosity was the same as these nominal values. We also note that there is a little elasticity taking place above 0.5% in weight percentage for the type chosen. However, this becomes ignorable at 0.1%, and treated as inelastic shear-thinning fluid in the later analysis of UVP data. For 0.1 wt% solution, the nominal viscosity is 10.8 mPas at 25 °C.

Uniform mixing of CMC in water is ascertained prior to experiments. The measurements are performed at Reynolds number in the range $50 < \text{Re} < 300$, where $\text{Re} = \rho DU/\mu_0$; here, μ_0 is zero-shear viscosity and U is the towing speed. U ranges from 20 to 100 mm/s in the present parametric study. The carriage is accelerated for initial 1–3 s after which the speed is constant and is decelerated again before stopping. The presented measurement is taken from few seconds after the carriage reaches uniform velocity, and thus, the flow at location measurement is not affected by initial acceleration. Towing experiment is done in stationary fluid; thus, uniform flow inlet velocity profile ahead of the cylinder is same as the towing speed of the cylinder. Influence of the upper and lower boundaries is negligible in present experimental condition because transducer is set at 3D (three times diameter) downstream. However, some influence might be felt in the wake sufficiently far from the cylinder, where momentum diffusion reaches the boundary. At 3D downstream, the cylinder symmetry is confirmed as is evident by the symmetry in measured velocity profile (shown later in Fig. 5) and resultant viscosity distribution that had a value close to unity outside wake until both boundaries (shown later in Fig. 7).

Spherical tracer particles (HPSS20, Mitsubishi Chemical), made of high-porous polymer with hydrophilic surface property to aqueous solution, are mixed as ultrasonic reflectors for UVP measurement. Density and diameter of the particles are 990 kg/m³ and 50–120 μm, respectively. The particles are dispersed homogeneously in the container.

2.2 UVP measurement system

UVP monitor model Duo (Met-flow S.A., Switzerland) was used for signal processing of ultrasound pulse echo waves. An ultrasonic transducer of basic frequency 4 MHz was used in vertical position for measuring velocity profiles in y direction. The transducer was fixed at $3D$ (3 times the diameter) downstream of the cylinder which is not close to cylinder so the three-dimensionality during vortex formation phenomenon can be avoided (Azmi and Zhou 2018). During each run, cylinder and transducer were towed together by the carriage at a constant controlled speed. The detailed specification of UVP system is given as below: maximum measurable range: 121.84 mm; sampling period: 123 ms; cycle per pulse: 4; spatial resolution: 0.74 mm; velocity

resolution: 0.477 mm/s; number of channels (velocity data points): 450; and number of velocity profiles stored: 450. The UVP spatial resolution did not change with the depth, and a constant sampling rate was adopted.

3 Algorithm development

Figure 4 shows the flowcharts for pressure estimation using velocity measurement data from PIV/UVP for Newtonian/non-Newtonian fluids. The pressure estimation process from PIV data is rather straightforward and has been established in literature. Here, it is shown with UVP-based algorithms for a clear comparison. Planar PIV measurement can provide velocity information of two components in two dimensions

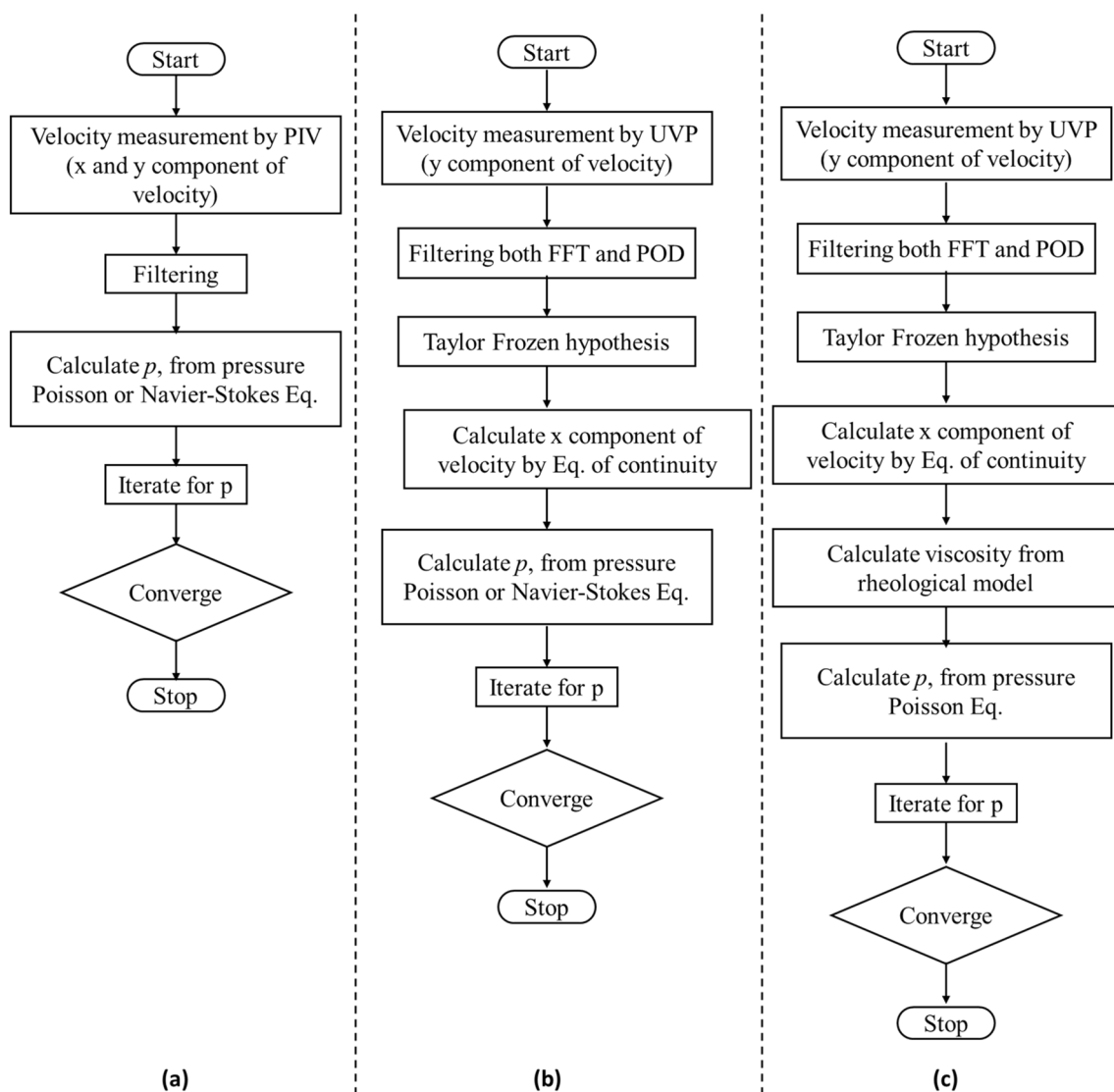


Fig. 4 Flowchart for pressure estimation from velocity measurement in the case of **a** transparent Newtonian fluids with PIV data **b** opaque/transparent Newtonian fluid with UVP data **c** opaque/transparent non-Newtonian fluid with UVP data

which is sufficient to estimate two-dimensional pressure field from Navier–Stokes or pressure Poisson equation. On the other hand, a UVP transducer can provide one component of velocity along the measurement line. The steps of pressure estimation in Newtonian fluid by PIV data and UVP data are shown in algorithms in Fig. 4a, b. In non-Newtonian fluids, the number of steps increases even more due to inclusion of rheological model as shown in Fig. 4c. These steps are further explained in following sections.

3.1 Velocity measurement

Measurement by UVP provides fluid velocity distribution in a spatiotemporal domain. Measured y component of velocity in the wake of cylinder is shown in Fig. 5a. Here, wake region can be clearly identified by observing alternating positive and negative regions of velocity. This shows the presence of regular vortices being shed in the wake as expected. This highlights the capability of present measurement in capturing flow structures. Before using this data in CFD equations, adequate filtering is required. Y and t are non-dimensionalized, respectively, by the diameter of cylinder D and D/U .

3.2 Velocity data filtering

Like PIV, velocity measurements by UVP also suffer from various noises. If proper attention is not rendered, these noises can magnify in calculation of second component of velocity and subsequently in viscosity and pressure. Combination of temporal (FFT, fast Fourier transform) and spatial (POD, proper orthogonal decomposition) filtering in UVP data had proved very efficient for noise reduction with conserving coherent flow structures as explained in our previous work (Tiwari et al. 2019). So, here we adopted the same filtering technique. After application of FFT low pass filtering, the velocity data are reconstructed as shown in Fig. 5b. Here, high frequency noises are filtered, and these

data are further analyzed with POD which is most efficient physics-based filter in removing non-organized errors (Charonko et al. 2010). POD (Sirovich 1987a, b) decomposes the experimental data in orthonormal basis functions and resolves the kinetic energy into POD modes. In present data, first mode captured the highest kinetic energy, 76% of total energy, which includes dominant structures of flow and modes 1 to 10 contain 99.5% of energy. In Fig. 5c, the reconstructed data after POD filtering are shown considering first ten POD modes. We applied FFT and POD combined filtering to all the results shown in this work. Importantly, using only FFT or only POD filtering for velocity data does not yield expected results for pressure profiles.

In the further sections, all governing equations are given in non-dimensional form. Following scaling variables, rendered dimensionless terms: d for length variables, U for velocity terms, ρU^2 for pressure. These equations are solved by finite difference method, and all the equations are discretized by central difference scheme.

3.3 Reconstruction of 2D velocity field

Velocity information in t - y plane is converted to x - y plane based on Taylor frozen hypothesis. The hypothesis combined with velocity measurement was found to be sufficient for pressure estimation with known convection velocity of dominant vortex structures (de Kat and Ganapathisubramani 2013) as

$$v(x, y) = v(x_0 - U_s \cdot t, y), \quad (3)$$

where U_s stands for the vortex migration velocity, which ranges from $0.55U$ to $0.65U$ relatively to the free stream velocity U (Lin and Hsieh 2003). x_0 represents the reference position of streamwise coordinates which is position of measurement. The 2D velocity is reconstructed by substituting the filtered v component of velocity in equation of continuity as

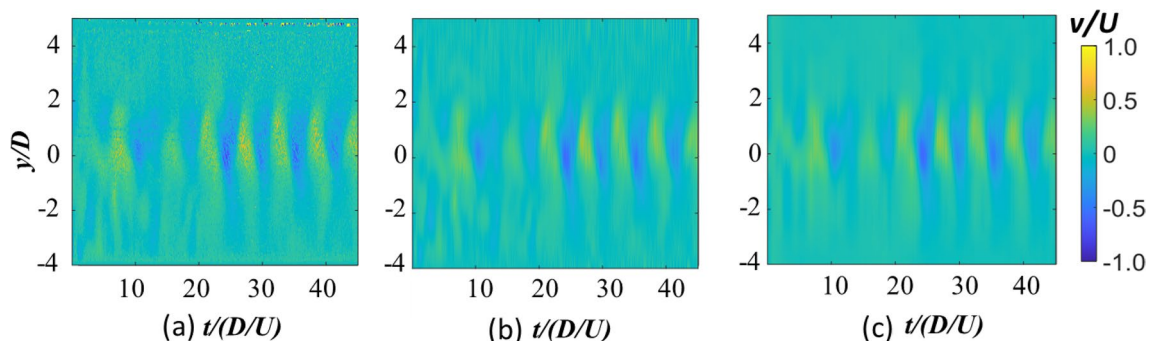


Fig. 5 Distribution of y component velocity measured by UVP: **a** original velocity data, **b** velocity after FFT filtering for noise reduction, **c** velocity after FFT and POD filtering considering the first ten POD modes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow u(x, y) = - \int \frac{\partial v(x, y)}{\partial y} dx. \quad (4)$$

Zero velocity is given in upper and lower boundaries, and the integration in multiple directions is applied to minimize the cumulative residual. The method is the same as our previous paper (Tiwari et al. 2019). The reconstructed 2D velocity vector field is shown in Fig. 6. Here, dominant vortices can be seen when only mode 1 of POD is considered for velocity reconstruction. When modes 1–3 are considered, very clear presence of staggered arrangement of vortices can be identified. Similarly, as more higher modes are considered cumulatively, smaller vortices appear along with staggered pattern.

3.4 Rheological model

As the fluid considered is non-Newtonian, proper selection of rheological model and its parameters are needed. In present study, low concentration of CMC (0.1% by wt) is used for which CMC chains are in their most extended form (Bayarri et al. 2009). The viscoelastic property of CMC was evaluated by creep test in study of (Benchabane and Bekkour 2008) and they observed that elastic behavior of CMC appeared only above concentration of 2.5% by weight. However, under certain flow conditions elastic properties has been observed at much lower wt% of CMC. Even our group found a hint of elasticity in 0.5 wt% aqueous CMC solution using USR (ultrasonic spinning rheometry), but in 0.1 wt%, such elasticity could not be measured. Experiments with 0.1% CMC solution were also reported by (Pinho and Whitelaw 1990) to measure drag reduction in pipe flow which is a manifestation of elastic effects. At Re of 5100, they found drag reduction of 2.4%, but at Re of 1480, the drag reduction was found to be negligible. In contrast at higher Re (> 8000), significant drag reduction (> 20%) was observed. Hence, the elastic effects are significant only for turbulent flows at higher Re. Since CMC solution used in present study is quite low in concentration and our flow is

at $Re \leq 293$, we have characterized it as shear-thinning fluid having purely viscous characteristics.

Based on this, we adopted Carreau–Yasuda model, to represent shear-thinning property of test fluid. It has benefit over power law model in representing accurate shear-thinning property for zero to infinite range of strain rate (Tiwari et al. 2021). For Carreau–Yasuda fluids, non-dimensional viscosity is formulated as:

$$\eta = \frac{\mu_{\infty}}{\mu_0} + \left(1 - \frac{\mu_{\infty}}{\mu_0}\right) \left[1 + \left(\lambda \left(\frac{II_{2D}}{2}\right)^{\frac{1}{2}}\right)^a\right]^{m-1/a} \quad (5)$$

Here, λ , a and m are the Carreau–Yasuda parameters. II_{2D} is the second invariant of strain rate tensor which can be written in case of two-dimensional incompressible flow as:

$$II_{2D} = \sum_i \sum_j \gamma_{ij} \gamma_{ji} = \gamma_{xx}^2 + \gamma_{xy}^2 + \gamma_{yx}^2 + \gamma_{yy}^2 \quad (6)$$

Here, γ_{ij} is strain rate tensor. The parameters for Carreau–Yasuda model were obtained from literature where rheometer experiments were performed using same solution as the one in present work at $\gamma = 10\text{--}4000 \text{ s}^{-1}$ (Coelho and Pinho 2003). The parameters are reproduced in Table 3. The representative viscosity of present 0.1 wt% aqueous CMC solution is 10.8 mPas (as noted in Sect. 2.1), and the representative viscosity for the solution used by Coelho and Pinho 2003 is 10.835 mPas (average of zero and infinite shear rate viscosity). These values are almost same. Moreover, literature shows that the flow curves obtained by two very different group of researchers for 0.1 wt% aqueous CMC solution are almost identical (varying within 5%). These data are shown in appendix (Fig. 16). This gives us the confidence

Table 3 Parameters of Carreau–Yasuda model

Solution	μ_0 (Pa s)	μ_{∞} (Pa s)	λ (s)	a	m
0.1%CMC	0.01367	0.0008	0.01317	0.6671	0.5864

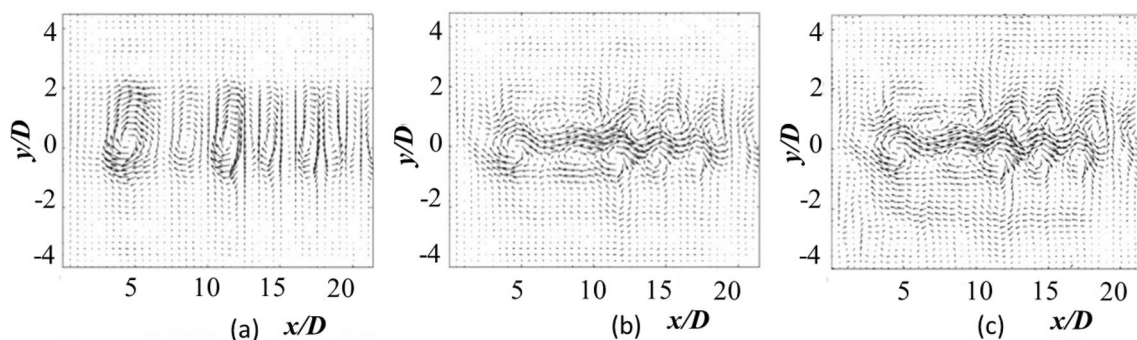


Fig. 6 Reconstructed velocity vector field considering POD **a** mode 1, **b** modes 1 to 3 and **c** modes 1 to 10

to believe that the rheological parameters for our test fluid will be very close to the one proposed by (Coelho and Pinho 2003) and is justified in current work.

In typical rheometer tests, the strain rate is varied and corresponding viscosity is measured. By repeating such experiment for a range of strain rate, a relationship can be found between strain rate and viscosity. In actual experiments, e.g., flow behind cylinder, a range of strain rate would appear depending on location and flow velocity. Still the local viscosity follows the same relationship with local strain rate as measured by controlled rheometer measurements.

3.5 Pressure estimation method

The dimensionless momentum equation can be written as

$$\frac{\partial p}{\partial x} = -\frac{\partial u}{\partial t} - \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{1}{\text{Re}} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right), \quad (7)$$

$$\frac{\partial p}{\partial y} = -\frac{\partial v}{\partial t} - \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) + \frac{1}{\text{Re}} \left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right), \quad (8)$$

where τ represents stress tensor excluding pressure. Relation between the stress τ and strain rate γ in two-dimensional flow is given by

$$\tau_{ij} = \eta \gamma_{ij} \rightarrow \begin{pmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{yx} & \tau_{yy} \end{pmatrix} = \eta \begin{pmatrix} \gamma_{xx} & \gamma_{xy} \\ \gamma_{yx} & \gamma_{yy} \end{pmatrix}, \quad (9)$$

where γ_{ij} is computed from measured velocity field as:

$$\gamma_{ij} = \begin{pmatrix} \gamma_{xx} & \gamma_{xy} \\ \gamma_{yx} & \gamma_{yy} \end{pmatrix} = \begin{pmatrix} 2 \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2 \frac{\partial v}{\partial y} \end{pmatrix}. \quad (10)$$

All the components of the tensor, Eq. (10), are obtainable on applying the continuity equation, Eq. (4), under the assumption of two-dimensional flow. Substitution of Eqs. (9) and (10) into Eqs. (7) and (8), we get

$$\frac{\partial p}{\partial x} = -\frac{\partial u}{\partial t} - \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{\eta}{\text{Re}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{2}{\text{Re}} \left(\gamma_{xx} \frac{\partial \eta}{\partial x} + \gamma_{yx} \frac{\partial \eta}{\partial y} \right), \quad (11)$$

$$\frac{\partial p}{\partial y} = -\frac{\partial v}{\partial t} - \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) + \frac{\eta}{\text{Re}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{2}{\text{Re}} \left(\gamma_{xy} \frac{\partial \eta}{\partial x} + \gamma_{yy} \frac{\partial \eta}{\partial y} \right). \quad (12)$$

The last four terms in these equations capture the dynamic effects due to non-Newtonian nature of fluid flow. After

taking divergence of the pressure gradient vector field, the pressure Poisson equation is obtained as

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\partial}{\partial x} \left(\frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial p}{\partial y} \right) \quad (13)$$

Unlike Newtonian fluid flows, the source term of pressure Poisson eq. in non-Newtonian fluid flows is a function of viscosity, so even after application of equation of continuity, various terms do not cancel out. Substitution of Eqs. (11) and (12) into Eq. (13) leads to a complicated source term of the Poisson equation, which produces higher order and mixed derivatives as below:

$$\begin{aligned} \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = & -2 \left(\frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \right) \\ & + \frac{1}{\text{Re}} \frac{\partial \eta}{\partial y} \frac{\partial}{\partial x} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) + \frac{1}{\text{Re}} \frac{\partial \eta}{\partial x} \frac{\partial}{\partial y} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \\ & + \frac{2}{\text{Re}} \left(\frac{\partial^2 \eta}{\partial x^2} \frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial x} \frac{\partial^2 u}{\partial x^2} \right) \\ & + \frac{2}{\text{Re}} \left(\frac{\partial^2 \eta}{\partial y^2} \frac{\partial v}{\partial y} + \frac{\partial \eta}{\partial y} \frac{\partial^2 v}{\partial y^2} \right) + \frac{2}{\text{Re}} \frac{\partial^2 \eta}{\partial x \partial y} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right). \end{aligned} \quad (14)$$

With limited spatial resolution of experimental data, it is impossible to directly compute all the higher order derivative terms in Eq. (14) accurately. So, for present pressure calculation, we adopt a two-step approach: right hand side of Eqs. (11) and (12) is computed from UVP data at all grid points with second-order finite differencing scheme, and then, these results are substituted in the right hand side of Eq. (13) to obtain source term of Poisson equation. Equation (13) is then solved for pressure distribution. This process provides two advantages: one is unnecessary of finite differencing for the second-order

mixed derivatives of velocity components that appears in the last five terms in Eq. (14). The other is elimination of

rotation in the pressure gradient vector field. Note that if spatial continuity of measured velocity data is lacking due to lower resolution, even Eq. (13) cannot provide a converged solution of pressure which can be the case with PIV data (Tiwari et al. 2021).

After applying Taylor frozen hypothesis and equation of continuity, the measurement domain was treated as computational domain for pressure estimation. The spatial resolutions in x and y directions are 1.13 and 0.74 mm, respectively. The boundary conditions of pressure are as follows: at left (upstream) and right (downstream) boundaries, Neumann boundary condition of pressure is applied (Eqs. (11, 12)). Zero pressure gradient conditions are applied at top and bottom boundaries. At four corners, Dirichlet condition of pressure is applied to control the absolute pressure.

4 Results and discussion

A demonstration of reconstruction of viscosity and pressure fields from UVP measurements and their relationship is discussed for $Re = 88$ in Sects. 4.1, 4.2 and 4.3. Then, a parametric study for Re in the range 58 to 293 is presented in Sect. 4.4.

4.1 Viscosity distribution

In Fig. 7, viscosity distribution in the wake of cylinder estimated with Carreau–Yasuda model is shown for three different cases corresponding to UVP data considering, POD mode 1, modes 1 to 3 and modes 1 to 10. POD mode 1 shows lowering of viscosity inside the wake region due to the shear-thinning property of aqueous solution of CMC. As higher POD modes are considered, additional low viscosity regions is observed in wake due to appearance of smaller eddies. Nearly 20% reduction of viscosity is observed at vortex centers. As the strain rates increase in vortex centers, the viscosity should decrease.

4.2 Pressure distribution

Once viscosity profiles are obtained, pressure distribution can be obtained after solving Eq. (13). Computed pressure distribution is shown in Fig. 8. Similar to the viscosity field, pressure distributions are obtained by considering POD mode 1, modes 1 to 3 and modes 1 to 10 of the velocity. The alternating low and high-pressure regions are observed in the wake region due to staggered arrangement of vortices. The presence of vortices can be easily distinguished by the local minimum of pressure. Here, the range of pressure

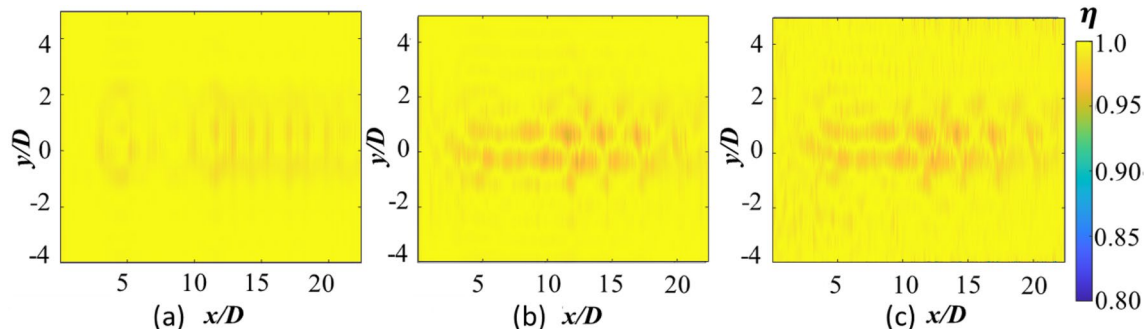


Fig. 7 Viscosity variation in computational domain for data corresponding to POD **a** mode 1 **b** modes 1 to 3 and **c** modes 1 to 10, respectively ($Re = 88$)

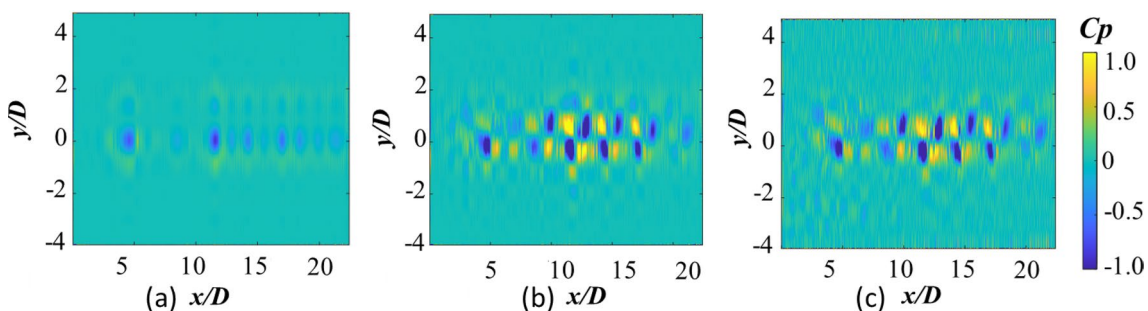


Fig. 8 Pressure distribution in computational domain considering velocity data of POD **a** mode 1, **b** modes 1 to 3 and **c** modes 1 to 10

coefficient, $C_p = p/\rho U^2$, between -1 to $+1$ is reasonable in cylinder flows which confirms the quantitative accuracy of the algorithm. To understand the influence of rheological model on pressure distribution, rheological parameters a and m are varied by 5% and pressure distribution is re-computed in Appendix Fig. 17. There is corresponding change in the C_p by 5%. This indicates that viscosity plays a non-negligible role in determination of pressure distribution. This is different than the case of usual Newtonian fluids where viscous term is often neglected during pressure estimation procedure. However, (McClure and Yarusevych 2017) noted in their work that viscous terms should be retained if Re is below 100. For the case of non-Newtonian fluids, the viscous term calculation is even more indispensable as it directly affects the flow dynamics. More work needs to be done to completely understand the role of viscous terms and specifically the parameters of rheological parameters for different non-Newtonian fluids in pressure distribution estimation.

4.3 Correlation of viscosity to enstrophy and pressure

In last sections, viscosity and pressure estimation was presented. Another scalar function roughly representing the

intensity of vortex structures is enstrophy. In this section, an attempt is made to understand the correlation between these scalars viscosity, pressure and enstrophy. First, phase averaging was performed in the range $7.5 \leq x/D \leq 17.5$ and $-2 \leq y/D \leq 2$ for pressure, and viscosity in the wake. In Fig. 9, distribution of phase averaged pressure and viscosity distribution are plotted excluding parts of the free stream region. The low-pressure regions corresponding to the high viscosity regions are indicated in squares in Fig. 9a, b which represents the inverse relation between pressure and viscosity. These relations can also be confirmed by performing a quantitative assessment with cross-correlation analysis. The correlations between pressure, enstrophy and viscosity were investigated for wake region by quantifying them using the following parameter.

$$Cc = \frac{1}{N-1} \sum_{i=1}^N \left(\frac{A_i - \zeta_A}{\sigma_A} \right) \left(\frac{B_i - \zeta_B}{\sigma_B} \right) \quad (15)$$

Here, Cc represents the linear dependence of two variables A and B , having N observations. ζ and σ are mean and standard deviation, respectively.

In Fig. 10, the viscosity variations with enstrophy and pressure are presented with correlation coefficient (Cc). The

Fig. 9 Phase averaged **a** pressure and **b** viscosity distributions in wake region

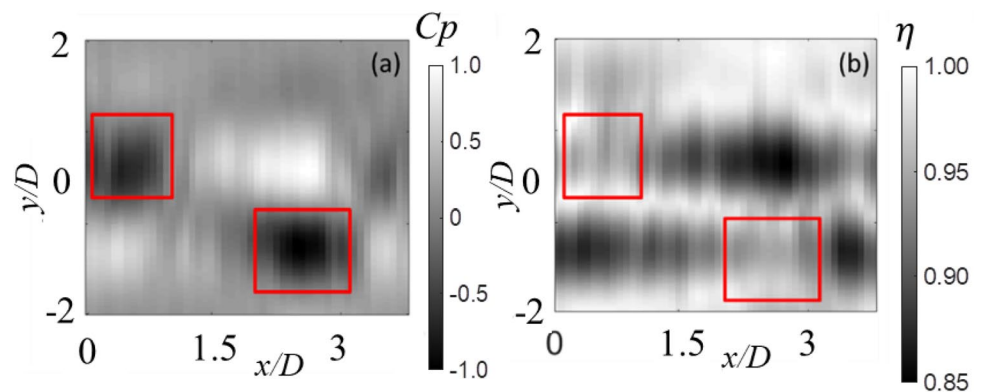
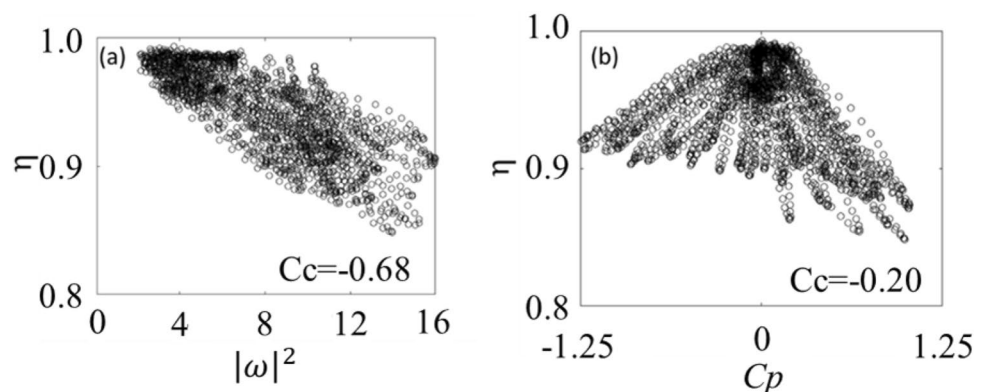


Fig. 10 Viscosity variation with **a** enstrophy and **b** pressure in computational domain



scatter plots of η vs $|\omega|^2$ (Fig. 10a) and η vs Cp (Fig. 10b) show that they have inverse relationship which is confirmed by negative sign of Cc . This is also in the support of visual inspection of inverse relation of pressure and viscosity as shown in Fig. 9a, b. Quantitatively, viscosity is more correlated with enstrophy than with pressure which means that the shear-thinning characteristic of fluid is more dependent on kinematics than the dynamics of fluid. This is expected because viscosity is estimated from velocity and its gradients via Carreau Yasuda model and enstrophy is also calculated from velocity gradients. However, the velocity gradients employed in their calculation are not identical. In Fig. 10b, the negative pressure region represents the interior of individual vortices, while positive pressure region represents the shear layer between vortices.

4.4 Parametric study for variable Re

A parametric study is conducted for flows at $Re = 58, 88, 146$ and 293 . This range of Re is targeted because stable two-dimensional vortex streets were formed in the wake in this range. The reconstructed 2D velocity vectors after considering POD mode 1 to 5 are shown for several Re in Fig. 11.

In Fig. 11d, the measurement time was almost half of previous cases. In Newtonian fluids, Strouhal number (St) of vortex shedding increases with Re (Williamson 1996)

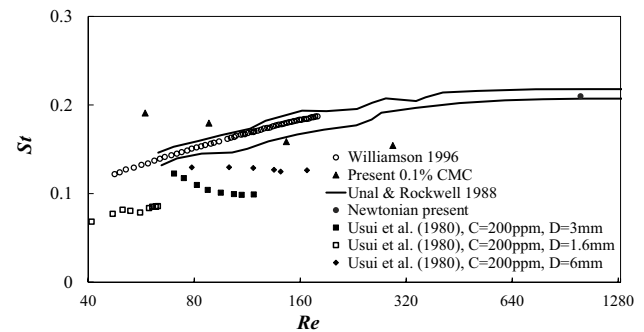
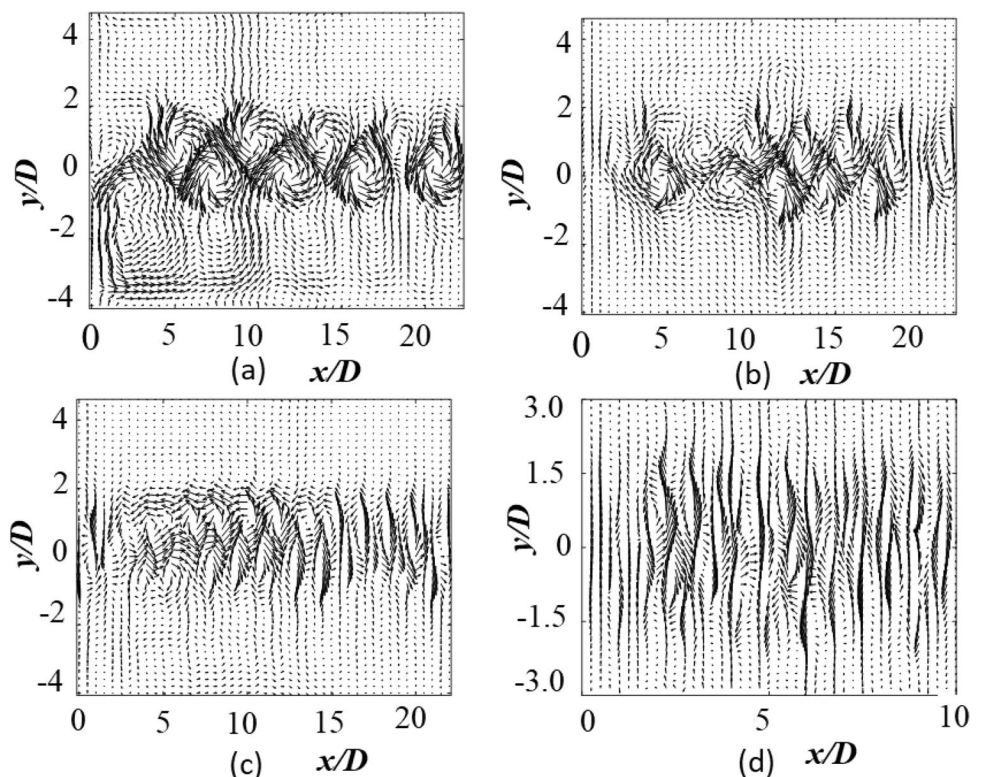


Fig. 12 Variation of vortex shedding frequency by Strouhal number with Re for Newtonian and non-Newtonian fluid flows

for the range of Re considered here (Fig. 12). However, in shear-thinning fluid flows we found that Strouhal number decreases with Re as shown in Fig. 12. For Newtonian fluid's von Karman wake at $30 < Re < 1000$, Strouhal number from literature is also plotted in Fig. 12. We have also included the St measured in present rig for water flow at $Re = 1000$ to show the accuracy of UVP system with equation of continuity in capturing wake dynamics. Interestingly for $Re < 100$, polymer additives are found useful to reduce the vortex shedding frequency (Kalashnikov and Kudin 1970). But as Re reaches $\sim 10^4$, the shedding frequency is not much

Fig. 11 Velocity vector plots obtained by UVP measurements filtered by FFT and POD for Re **a** 58, **b** 88, **c** 146, **d** 293



different from the Newtonian fluid flows (Sarpkaya et al. 1973). Decrement of Strouhal number with polymer concentration, diameter of cylinder and Re was observed in experimental study on flow over cylinder in viscoelastic fluids by (Usui et al. 1980). In Fig. 12, we have included their results obtained for polyethylene oxide (PEO) of concentration of 200 ppm and three cases of varying diameters of 1.6 mm, 3 mm and 6 mm. In case of diameter 1.6 mm, they have found increasing trend of St and as diameter increases further to 3 mm decrement trend of St observed. Hence, the decrement in St for non-Newtonian fluids is not unexpected and can be observed under various flow configurations. The PEO solutions are now known to be elastic so that decrease of St measured by Usui et al. (1980) can be attributed to combination of shear-thinning with elasticity. However in our CMC solution, we think that shear-thinning effect is dominant over elastic effects and as such the decrement in St can be mostly due to shear-thinning effect.

Figure 13 shows viscosity distribution in the wake for various Re . It is observed that the reduction in local viscosity is greater inside the vortex street as Re increases. This means that effective Re inside the street further increases with nominal Re because Re is inversely proportional to viscosity. Hence, decrease of St in the shear-thinning fluid, which we found in present UVP measurement, cannot be explained only by the concept of effective Re . (This is because St increases with Re in Newtonian fluid flows.) It

should be discussed based on sustainability of vortex structures that is a matter of coupling between viscosity (which decides viscous force) and pressure around individual vortices. Due to lower viscosity in the wake, the sustainability of vortex structures increases, i.e., the vortices stay for the longer period by the virtue of increased inertial forces. So, the vortex shedding frequency may not increase as much it increases in flow of a corresponding Newtonian fluid.

Velocity and viscosity data obtained and shown in Figs. 11 and 13 are substituted in Eq. (13) to determine the pressure distribution. Pressure distributions for various Re are shown in Fig. 14. The alternating positive and negative pressure spots are accurately reconstructed in wake region for all Re which indicates the applicability of the new proposed method using pressure Poisson equation in unsteady non-Newtonian fluid flows.

5 Conclusion

Pressure estimation algorithm was proposed for UVP-based measurement in shear-thinning fluid flow in cylinder wake. Velocity data obtained by UVP were substituted in equation of continuity, rheological model and pressure Poisson equation to explore local distributions of viscosity and pressure. This sequence successfully estimated the pressure distribution that couples with viscosity fluctuation inside the vortex

Fig. 13 Distribution of viscosity obtained by substitution of velocity data in Carreau–Yasuda model for Re **a** 58, **b** 88, **c** 146, **d** 293

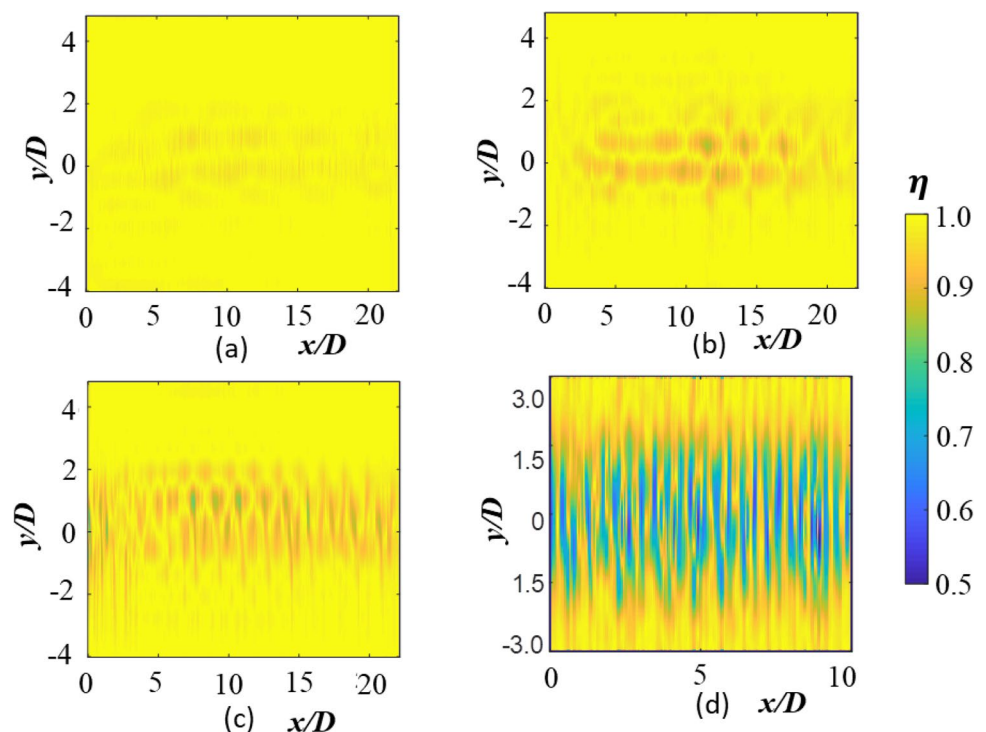
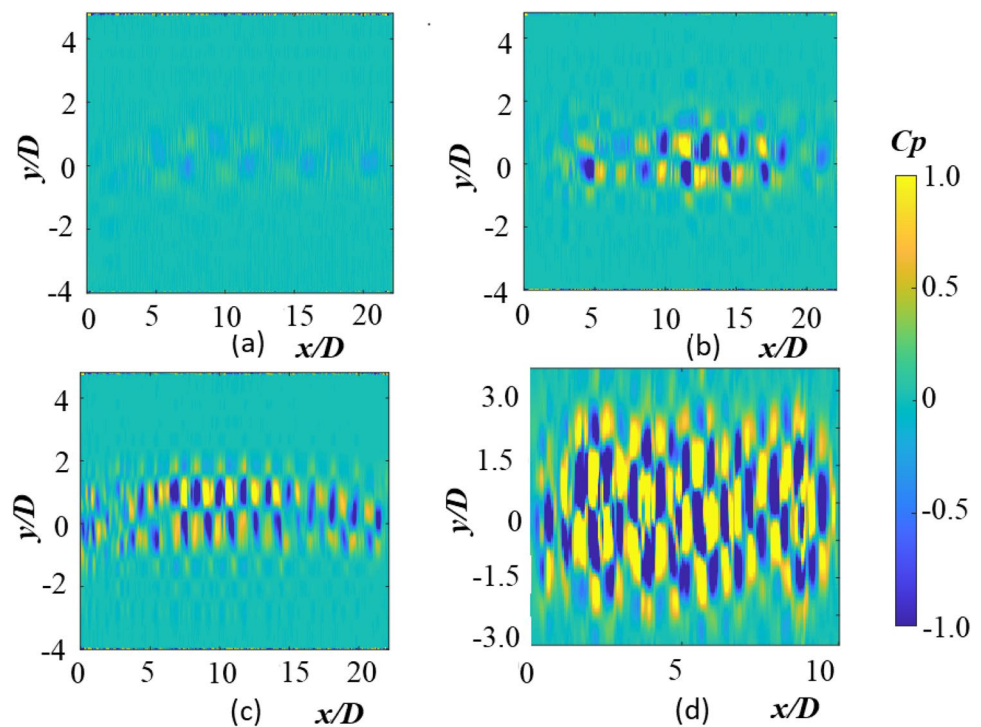


Fig. 14 Distribution of pressure for Re **a** 58, **b** 88, **c** 146, **d** 293



street. Parametric study was conducted for Re in the range 55–293. In our analysis, we have found that the Strouhal number decreases with Re up to 293 due to increment of vortex sustainability which causes the longer stay of vortices in wake. Capability of UVP measurement to capture the crucial flow structures and dynamical parameters was successfully demonstrated. It provides a tool to study the flow dynamics of more complex non-Newtonian fluids in conjunction with usual rheometer experiments.

Appendix

CFD simulation of the same flow

Since there is no alternative measurement technique available for validation, we performed a numerical simulation on the same conditions as the experiment using finite volume solver. We applied the Carreau–Yasuda model with same parameters as used in present work. A good agreement can be confirmed for the pressure field inside the wake. In this simulation, second-order finite difference spatial discretization scheme and a second-order backward Euler implicit time marching scheme were used. The equations were

discretized and solved on a two-dimensional unstructured mesh for $Re = 146$ ($U = 50$ mm/s). Total number of nodes were 30,000 in computational domain of size $40DX40D$. Velocity and pressure fields are shown in Fig. 15.

Here, v component of velocity varies between -0.6 to 0.6 (v/U) which is similar to our measurement shown in Fig. 4. Alternating low-pressure regions represent the staggered vortices in the wake which were successfully reconstructed similarly to Fig. 10c. C_p data extracted at a point at 3D downstream of cylinder at center line of computational domain and shown in Fig. 15d. In the pressure distribution plot, pressure coefficient varies between $(-1$ to $1)$, which is similar in our case as shown in Fig. 14c.

Uncertainty propagation of rheometry into dynamics

In present study, rheological parameters are referred from rheometer of (Coelho and Pinho, 2003). Here, we attempt to evaluate the error propagation in estimated pressure due to uncertainty present in rheological parameter. In literature, we have found that the rheological parameters of CMC (0.1% concentration) change only slightly (less than 1%) among published results of Coelho and Pinho (2003), Benslimane et al. (2016, 2018) due to different

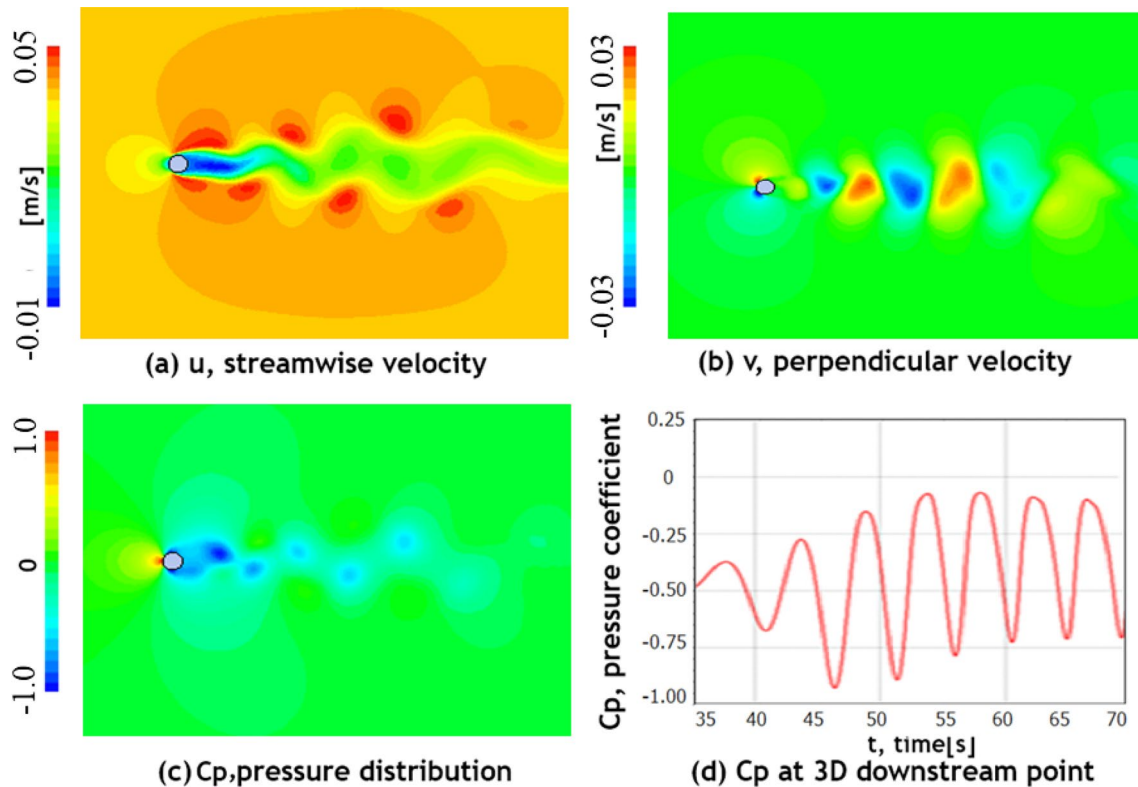


Fig. 15 Result of CFD simulation at $U=50$ mm/s using Carreau–Yasuda model with same parameters as used in present measurement. **a** Streamwise velocity, **b** perpendicular velocity, **c** pressure distribu-

tion and **d** pressure fluctuation due to vortex shedding at 3D downstream the cylinder on the central axis

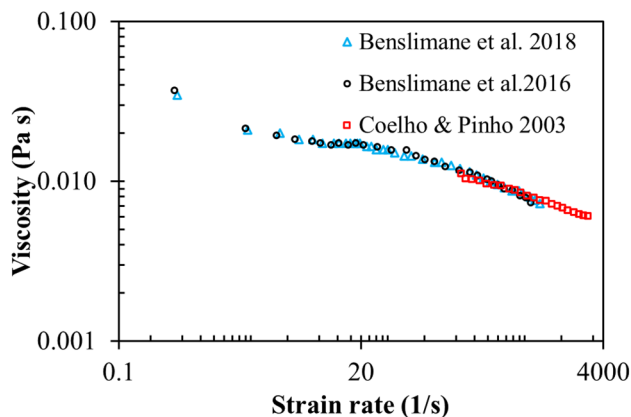


Fig. 16 Comparison of viscosity and strain rate flow curve obtained in three different CMC samples

manufacturing company or water quality or different rheometer. Here, three flow curves are plotted together for aqueous solution of CMC of 0.1% concentration for comparison. This is shown in Fig. 16.

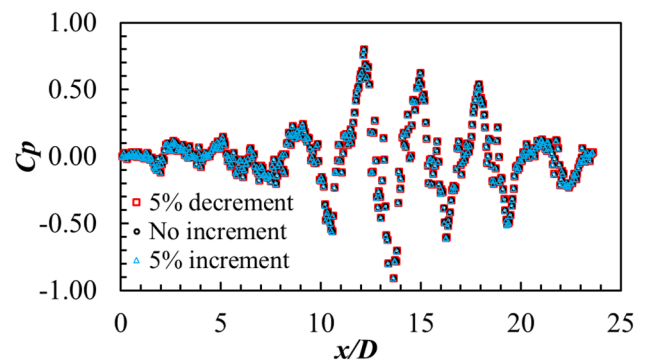


Fig. 17 Pressure variation at the center of domain for different cases of increment/decrement of rheological parameters

To account for some variation, we altered the rheological parameters by 5% and recalculated the pressure profiles. In Fig. 17, pressure profiles are obtained at the center line of measurement domain. The reconstructed pressure profiles changed under 5% in both increment/decrement cases.

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