Regime diagram of thermal convection in liquid metal with horizontal magnetic field

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We performed laboratory experiments of Rayleigh-Benard convection by using liquid gallium, under various intensities of a uniform horizontal magnetic field. An ultrasonic velocity profiling method was used to visualize the spatiotemporal structure of the flow. The range of Rayleigh number (Ra) is from critical value to 10^5, and that of Chandrasekhar number (Q) is 0-1100. A regime diagram of convection patterns was established in relation to Ra and Q, for a square vessel with aspect ratio five. We recognized five flow regimes; (I) fluctuating large-scale pattern without roll, (II) weakly constrained roll with fluctuations, (III) continuous oscillation of roll, (IV) repetition of roll number transitions with random reversals of the flow direction, and (V) steady 2-D rolls. These flow regimes are classified by the value of Ra/Q, that is the ratio of buoyancy force to the Lorentz force. We successfully reproduced these five flow regimes by numerical simulations. The process of the flow reversal is clarified in detail by the simulation.

Keywords: Rayleigh-Benard convection, liquid metal, horizontal magnetic field, flow regimes

1 INTRODUCTION

The study of the nature of thermal convection in liquid metals under a magnetic field is important to understand the dynamics of planetary metallic cores and in engineering applications such as stable crystal growth and enhancing the efficiency of heat exchangers. An electric current is induced when a flow of liquid metal crosses a magnetic field and it generates a Lorentz force. The Lorentz force changes the force balance, making the flow behavior different from flows in situations without magnetic fields. Generally, the viscosity of liquid metals is very low and their flow easily becomes turbulent, but when a magnetic field is applied to liquid metals, it enhances two-dimensionality and creates anisotropic flow structures suppressing turbulence, depending on the direction and intensity of the field [1].

A typical setting to study the effect of a magnetic field B on fluid flow is a thermal convection system driven by a vertical temperature gradient, that is, Rayleigh-Benard (R-B) convection. The controlling non-dimensional parameters in a R-B convection system under an imposed uniform magnetic field are the Rayleigh number (Ra), and the Chandrasekhar number (Q) [2]. In R-B convection of low Pr fluid like liquid metals, the region where two-dimensional rolls remain steady in the wave number-Rayleigh number space (the Busse balloon) is limited by the Eckhaus instability on the smaller wave number side, by the skewed-varicose instability on the larger wave number side, and by the oscillatory instability on the larger Ra values [3]. With a uniform horizontal magnetic field imposed, the Busse balloon extends by the increase of the onset Ra of oscillatory instability [4]. The delay of the onset of oscillation toward higher Ra with horizontal B is confirmed by laboratory experiments [5-7].

However, these studies have not fully elucidated the style of convection in Ra regions beyond the breakdown of steady roll structures, because of the lack of information on flow patterns. Most laboratory experiments have been based on point measurements of the temperature in liquid metals. Temperature measurements are suitable for monitoring local fluctuations with high resolutions, but cannot capture global patterns and sizes of flow structures. The most Important problem needing elucidation is the structural developments in the transition to chaos and turbulence. Observation of the spatio-temporal variation of flow is essential for this purpose. Here we utilized ultrasonic velocity profiler to illuminate the internal flow structure of a liquid metal.

2 APARATUS AND SETTING

The vessel we used has a square geometry with aspect ratio five (Fig. 1) [8,9]. The top and bottom plates are made of copper, and the temperature of each plate is maintained by circulating water. Liquid gallium is used as the working fluid. Transducers for the Ultrasonic Velocity Profiler are set in holes in the Teflon sidewalls, and are in direct contact with the liquid gallium. We are using UVP-Duo (Met-Flow S.A.), with the basic frequency of 4 MHz. The flow velocities of the gallium were measured along four lines from the transducers (uv1-uv4). The UVP measures the projected flow velocity along each line. The temperature fluctuation in the gallium layer is monitored by a thermistor probe at the middle depth. We used a Helmholtz coil system to apply a uniform magnetic field. The direction of magnetic field is horizontal in this study, and its intensity is controlled by an electric power supply. The spatial variance in the magnetic field is within 2 % around the vessel.

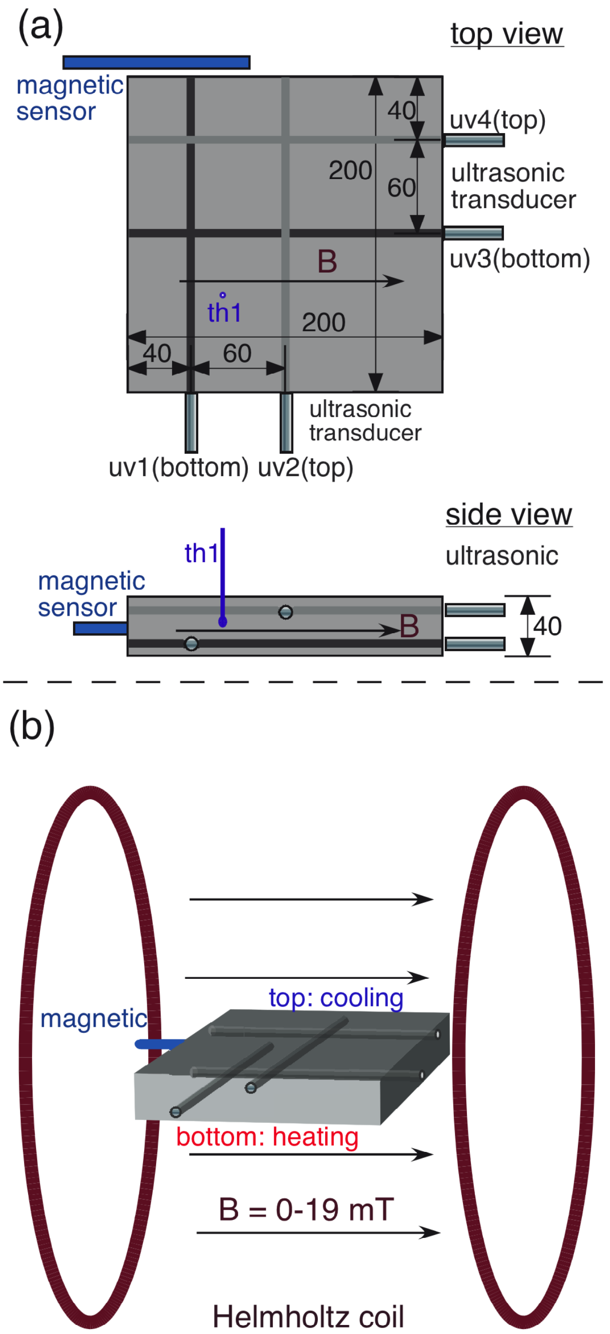

Figure 1: (a) Geometry of the vessel and measurement lines with ultrasonic beam. Liquid gallium is filled in the vessel. Temperature fluctuation is monitored by the probe th1. The numbers are the dimensions in mm. (b) The vessel in the Helmholtz coil system. Horizontal magnetic field B is applied.

3 RESULTS

We identified five flow regimes depending on Ra and Q, using the flow velocity information by the UVP. Regime (I) is a state with no roll structures (or fluctuating large-scale cells), similar to the state with no imposed magnetic field. Regimes (II)-(V) are states showing an effect of the magnetic field with roll-like structures with mean axes parallel to B. Regime (II) is a state with weakly constrained rolls in which the roll number transitions between 4 & 3 are observed. Regime (III) is a state with continuous oscillations with a 4-roll structure. Regime (IV) is a state where the roll number transitions between 5 & 4 occur intermittently, and reversals of flow direction in rolls are observed sometimes accompanied by the transitions. Regime (V) is a state with a steady 2-D roll structure, (V-a) with 5-rolls, and (V-b) with 4-rolls.

3.1 Examples of five flow regimes

Typical examples of the five flow regimes are shown in Figure 2.

Figure 2 (a) is a steady 5-roll structure with the roll axis fixed in the direction of the magnetic field (regime (V-a)). The flow parallel to B (uv3) is very weak. The patterns observed by the other lines uv1 and uv4 conformed to this, showing that the flow has a highly regular two-dimensional steady roll structure. This represents a typical flow regime under a strong horizontal magnetic field. There are five rolls showing that the aspect ratio of a roll is ~1.0, which approximates the most likely aspect ratio near the critical Ra with a no-slip boundary condition.

Figure 2 (b) is classified into regime (III), where the pattern oscillates periodically but it maintains a 4-roll state. The period of oscillation is nearly equal to the circulation time of the flow for a roll (~30 s).

Figure 2 (c) is classified into regime (IV). The flow velocity in the direction perpendicular to B is much larger than in the direction parallel to B, which shows the existence of a roll-like structure, while transitions in the roll number between 5 and 4 occur at irregular time intervals. Reversals of the flow direction in the rolls sometimes occur accompanied by the roll number transitions.

Figure 2 (d) is classified into (II), showing roll number transitions between 4 and 3 for the uv2 profile. The pattern maintains a roll-like structure, but the three-dimensionality of the flow has increased. Flow reversals are not clearly observed in this regime.

Figure 2 (e) is classified into regime (I), where whole- or half- vessel scale flows are observed with short time fluctuations and the magnitude of the velocity is similar in both directions. There are no roll-like structures, though the experiment took place with a horizontal magnetic field imposed. The behavior is similar to that without a magnetic field.

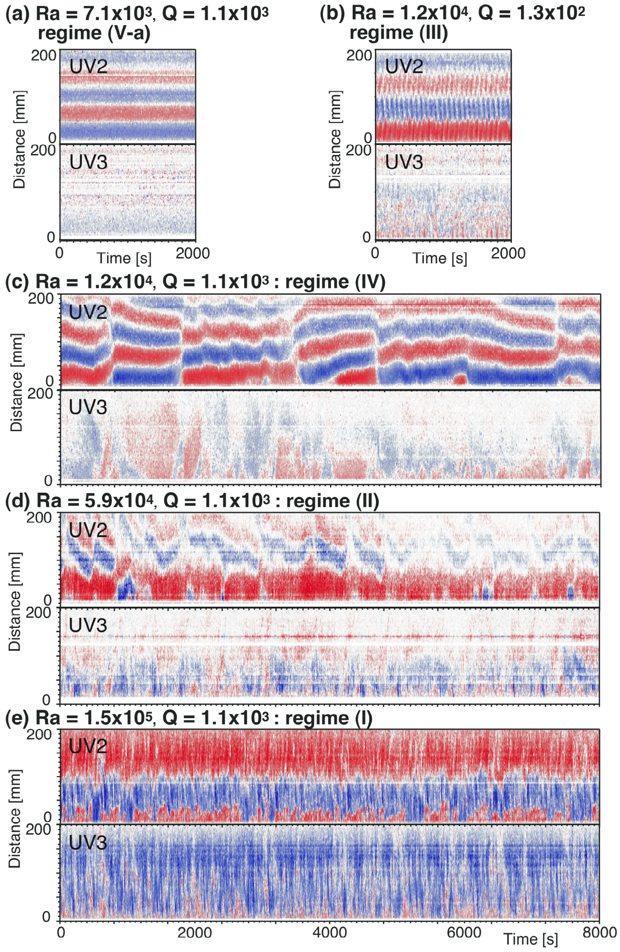

Figure 2: Typical examples of five flow regimes (I-V). Time-space maps of the horizontal flow velocity for the lines uv2 and uv3 are displayed. The direction and magnitude of the velocity are shown in the color scale, red is away from the transducer (distance=0) and blue is toward it.

3.2 Regime diagram of convection patterns

Figure 3 summarizes a regime diagram of the convection behavior in the Q-Ra plane, for this square aspect ratio five vessel. On the regime diagram, isolines of the parameter ratio Ra/Q are also shown. These lines suggest that the classification of flow regimes above Ra~10^4 can be understood by the values of Ra/Q; the boundary line for regime (I) (no-roll regime) corresponds to Ra/Q~100, and the regime (IV) (roll number transition between 5 & 4, including flow reversals) is divided by the two lines Ra/Q~30 and Ra/Q~10. From the definitions of Ra and Q, Ra/Q means the ratio of the buoyancy to the Lorentz force. Hence the upper left side of the diagram is the region where the buoyancy is much larger than the Lorentz force, while the lower right side is the region where the Lorentz force is dominant. In the region where Ra/Q > 100, the effect of the applied magnetic field is not visible on the flow behavior; isotropic structures are dominant and the flow is in the developed turbulence.

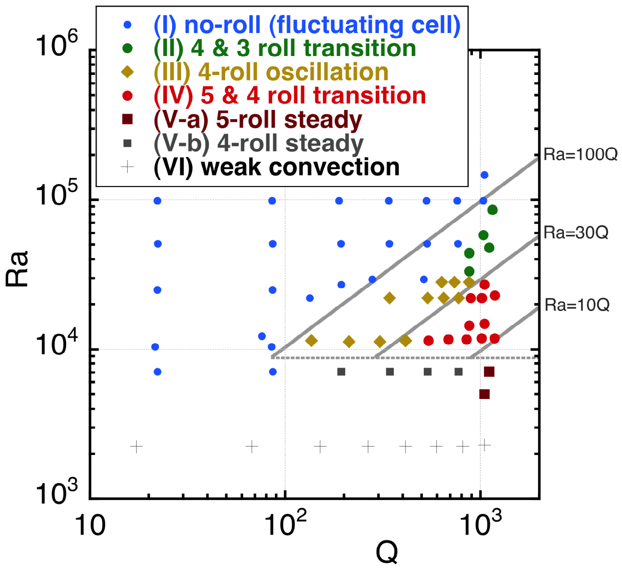

Figure 3: Regime diagram of the convection behavior on the Q-Ra plane. Oblique lines are the isolines of the parameter ratio Ra/Q, approximately corresponding to the regime boundaries.

3.3 Numerical simulation

We performed numerical simulations for the same setting as these laboratory experiments, with horizontal magnetic field imposed on the vessel of no-slip velocity boundaries. The five flow regimes mentioned above are reconstructed successfully in the simulations. The features observed by the velocity profiles as Fig. 2 are consistently reproduced for the same parameter sets of Ra and Q, and a regime diagram similar to Fig. 3 can be drawn.

The most interesting behavior is the reversal of the flow direction that characterized regime (IV). Here we present the three-dimensional detailed process of a flow reversal in Figure 4. The parameter setting for this case is Ra=1×10^4 and Q=1×10^3. The flow pattern keeps 5-roll structure for most of the duration, while reversal of flow occurs with random intervals. The Q_{3D} criterion [10] is used to illuminate the roll structures and their transition. At the thermal diffusion time t=40.1, the pattern shows 5-roll structure with its axis parallel to the magnetic field; the direction of circulation is clockwise for the rolls located close to the right and left sidewalls and at the center. These roll structures bend horizontally at t=40.8, with the rolls at frontal half are moving toward right, and the roll at the right sidewall is shrinking. Bended 4-roll structure is observed at t=41.3, and the flow velocity is much smaller than that of five-roll structure. Then, reconnection of the rolls between front and back occurs, and aligned 4-roll structure emerges for a short period. A new roll is growing along the left sidewall at t=42.0, and 5-roll structure is reproducing. The thermal diffusion time unit one corresponds to 130 s in the laboratory experiment.

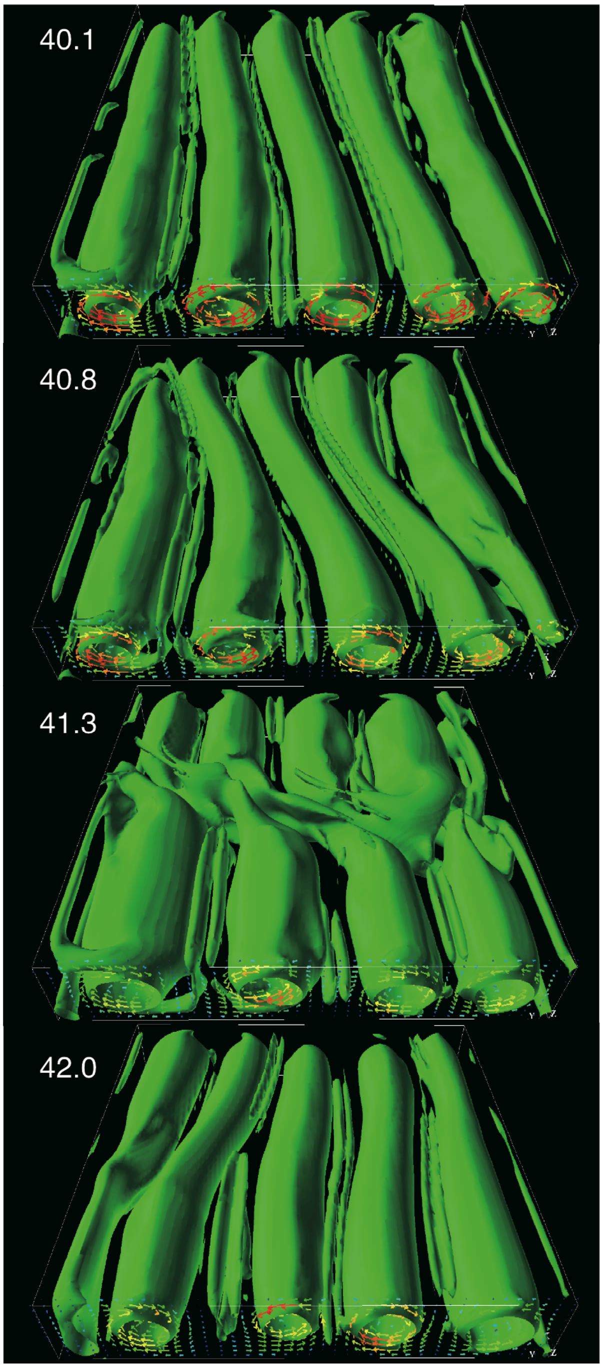

Figure 4: A process of flow reversal reproduced by a numerical simulation. The isosurface of Q_{3D}=0 is shown, with the flow near the x=0 wall is displayed by arrows.

4 DISCUSSIONS

A regime diagram of convection patterns under a horizontal magnetic field is established in relation to Ra and Q. As the relative intensity of the buoyancy increases, spatial and temporal variations in the pattern emerge from for Ra > 10^4. The increase in the horizontal scale with Ra causes incongruities in the roll size with the fixed geometry of the vessel. The discrepancy between the intrinsic roll size and the vessel size may be a factor in inducing the repeating transitions in the number of rolls and flow reversals (regime (IV)). At the Ra where the intrinsic roll size with fluctuations matches the vessel, the oscillatory state of the 4-roll structure is maintained for a long period (regime (III)). For higher Ra, a structure with a smaller wave number (3-roll) arises, with increased velocities in the direction parallel to B, and the roll-like structure becomes less clear (regime (II)). For Ra > 10^5, an almost isotropic flow is attained with three or two velocity clusters in both directions (regime (I)).

Transitions between the regimes observed here can be understood based on the Busse balloon concept. The steady roll in regime (V) locates inside of the Busse balloon, which may be extended to Ra~8000 by the imposed horizontal magnetic field. The oscillation of the 4-roll structure (regime (III)) corresponds to the traveling wave convection [11] over the Ra at the oscillatory instability boundary. Regime (IV), the repeated transitions between 5 & 4 rolls, locates adjacent to the region of steady and oscillatory rolls, which suggests that it has developed as a result of the skewed-varicose instability of the 5-roll steady structure.

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