Ultrasound Doppler rheometry from spin response of viscoelastic and bubbly Liquids

Yuichi MURAI, Yuji TASAKA, Kosuke SAKURAI, Kazuya OYAMA, and Yasushi TAKEDA
Faculty of Engineering, Hokkaido University, N13-W8, Kita-ku, Sapporo, 060-8628, Japan
(e-mail: murai@ring-me.eng.hokudai.ac.jp)

Ultrasound Doppler technique for velocity profile measurement is applied for the estimation of rheological properties such as viscosity, elastic module, yield stress as well as effective and complex shear viscosities. Scanning of transient velocity profiles on a single ultrasound measurement line allows the measurement to complete the shear strain rate - shear stress characteristics only by a single run of spinning fluids inside a cylindrical container. The rheometrical performance is evaluated by several test cases for viscoelastic liquid (PAA), and applied for the measurement of those properties in bubbly liquid to seek the viscoelastic response in Newtonian-gas-liquid combination. The results for bubbly liquid in transient spinning response show that the effective shear viscosity increases high when bubbles are stretched with the shear stress. This trend is totally different from the effective viscosity of bubbly liquid in equilibrium deformation such in a steady Couette flow.

Keywords: Ultrasound rheometry, Spin response, Ultrasonic velocity profiling, Viscoelasticity

1 INTRODUCTION

Liquid foods have an extremely wide range in rheological properties. Food rheology is thus positioned in high priority field in a part of life science. One concern that tightly relates to accident is the behavior of visco-elastic liquid food. These liquids sometimes cause unexpected resistance in swallowing them, and also worsen the quality control of food manufacturing process. In order to estimate these visco-elastic properties in liquid foods, a high-accurate but convenient method is desired to build up. Conventional methods for estimating them utilize indirect information obtained from differential pressure in a tube (Poiseulle viscometry), torque caused by the wall shear stress (torsional Couette viscometry), or terminal velocity of sphere (Stokes viscometry). In these methods, the measurement of global properties needs time-consuming work for changing the shear rate repeatedly. Handling of thixotropic liquid becomes difficult when such a time-marching measurement is adopted. In addition, these indirect measurements cannot provide any information in terms of internal structure of the flow. This problem slows down the scientific understanding of the rheological behavior of liquid food.

Since ultrasonic velocity profiler (UVP)[1] can acquire the velocity profiles as a function of time, it has a potential to become a high-performance rheometry. Because of ultrasonic principle, most of liquid foods can be measured regardless of the opaqueness, such as milk, yogurt, chocolates, and foam-shaped food. In this study, our objective is set to develop such a UVP-base rheometry by choosing the best flow configuration. The best configuration is call so when the spatio-temporal two-dimensional velocity information is fully utilized to determine the liquid properties. We propose here a transient shear response of fluid driven by spinning in a container. This flow has rich velocity fluctuation both in space and in time, and hence the wide relationship between the shear stress and the shear rate is acquired.

In this paper, we describe about the method of UVP-based rheometry, and the verification using polyacrylamide (PAA) solution. The internal structure of fluid deformation is also visualized from the UVP data for PAA solution. The present method is applied for experimental detection of viscoelastic property in bubbly liquid.

2 FLOW CONFIGURATION

Figure 1 shows general description of the present flow configuration. Comparing with pipe flow[2-5] the spinning flow enables one to acquire the transient shear response from quiescent state. Therefore, wall-clinging effect for the elasticity, fluidized behavior departing from the yield point, and shear-thinning effect are all captured simultaneously with a single run of the measurement. Furthermore, any liquid reaches at the rigid rotation in steady state so that the transient region is easily judged from the radial velocity profiles. Furthermore, temperature and pressure conditions are controlled easily because of packed liquid in the container. Detailed dimensions are as follows. The UVP measurement line is set at the distance of e=7mm from the central axis of the container which is made of acrylic resin with D=150 mm and 2 mm in thickness. Temperature is controlled to be constant at 20 deg C at atmospheric pressure. An ultrasonic absorber is put on the backside of the transducer. The basic frequency is set at 2 MHz. High-porous polymer
particles (HP20SS) are mixed to be ultrasonic reflector in target fluid. The speed of sound for PAA solution is 1485 m/s. Sampling period is 20 msec, and the number of profiles is 1024. Spatial resolution is 0.75 mm while the number of channels is 230. The flow velocity resolution is 1.5 mm/s. The rotational speed of the container can vary from 50 to 150 rpm.

Figure 1: Measurement line of UVP for liquid filled in circular container. $x$: distance from ultrasonic transducer along the path of ultrasonic pulse. $e$: distance from the center axis of the container to the nearest point on the measurement line. $D$: internal diameter of the container.

3 SUDDEN SPINNING RESPONSES

Three types of spinning response are investigated for 1wt% PAA solution. Comparing these three flow structures, we deduce which response the best reflects to the flow for estimating the rheological properties.

Figures 2, 3, and 4 represent spatio-temporal velocity measured by UVP for three different spinning modes; spinning up, spinning down, and impulsive spinning, respectively. In all figures, the velocity shown there is the circumferential component that is decomposed from the on-beam velocity along the measurement line of UVP. The method of the decomposition is reported in our previous paper[6].

On the spinning up mode shown in Figure 2, the circumferential velocity takes the largest value at $t=0.6s$ and it oscillates a few times before reaching the rigid rotation. This oscillation comes up due to elastic property of PAA solution and appears more obviously on the spinning down mode shown in Figure 3. On this mode, flow orientation changes repeatedly after the container is suddenly stopped. Calculating the damping ratio of velocity amplitudes during single cycle enables the average viscosity to be estimated while the oscillation frequency can be used for determining the system elasticity of liquid in this flow configuration.

These two responses are combined when impulsive spinning is given to the fluid as shown in Figure 4. The impulsive spinning is defined so when the quiescent fluid experiences instantaneous spinning within 0.5 second. On this mode, rigid rotation does not appear in the response.

By integrating the velocity profiles in time, the actual fluid deformation is visualized as shown in Figures 5 to 7. Among a number of representations, two cases are shown here: one is particle tracers being originally arranged in radial direction, and another is mesh tracers being square shape in the initial condition.

Figure 5 shows the fluid deformation on spinning up mode. The initial particle arrangement changes after the flow reaches the rigid rotation because of yielding of the fluid near the container wall. In contrast, the fluid in the core keeps the radial arrangements for pure elastic response there. Figure 6 shows the same visualization on spinning down mode. The particles are scattered inside the layer near the container wall while they keep the original arrangement in the core. As seen in mesh pattern, the core part is more rigidly kept its original
square pattern. Thus, the spinning down mode provides radial stratified structure stronger than the spinning up mode. Figure 7 shows the deformation on impulsive spinning mode. Obviously the particle displacement is limited in a shallow layer near the wall, and the mesh structure is unmodified in comparison with the other two spinning modes.

For visco-elastic liquid, the following Herschel-Bulkley model suitably expresses the shear stress.

$$\tau = E\gamma = E \left[ \frac{\partial v}{\partial t} + \frac{u}{r} \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau \right)$$  \hspace{1cm} (3)

$$\tau = K \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right)^n + \tau_y \quad (\tau \geq \tau_y)$$  \hspace{1cm} (4)

Four independent constants involved in the model are determined from velocity information based on least square approach. The cost function $g$, which should be zero when the model completely matches the properties of actual fluid, is defined as follows.

$$g(K, E, n, \tau_y) = \left( \rho \frac{\partial v}{\partial t} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau \right) \right)^2$$  \hspace{1cm} (5)

We call $g$ as cost function because dimension of the error assessment is arbitrary. In the case of Eq.(5), the dimension of $g$ is squared force per unit volume, i.e. N/m$^3$ or Pa/m. Fully utilizing the spatio-temporal velocity information, the global cost function is defined as

$$G(K, E, n, \tau_y) = \int g(K, E, n, \tau_y) dr dt \rightarrow \min$$  \hspace{1cm} (6)

Here the four constants are determined by random search algorithm to find out the condition that $G$ gets minimum value. Using Eq.(6), the least value of $G$ indicates the discrepancy of the employed rheological model from the actual property of the fluid. In this sense, the present method can assess the validity of the rheological model quantitatively.

![Figure 5: Distortion of fluid in PAA solution on spinning up mode. Left: particle tracers; right: mesh tracers.](image)

![Figure 6: Distortion of fluid in PAA solution on spinning down mode. Left: particle tracers; right: mesh tracers.](image)

![Figure 7: Distortion of fluid in PAA solution on impulsive spinning mode. Left: particle tracers; right: mesh tracers.](image)

## 4 RHEOLOGICAL PROPERTIES

The following equation of circumferential momentum conservation is satisfied for any type of fluid when the internal shear stress is described as $\tau$.

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{uv}{r} \frac{\partial v}{\partial \theta} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau \right)$$  \hspace{1cm} (1)

Here $\rho$ is density, $t$ the time, $v$ the circumferential velocity, $u$ the radial velocity, $r$ the radial coordinate, and $\theta$ the angle. The velocity $v$ is obtained by UVP. When the radial velocity is negligible and the flow keeps axisymmetric structure, Eq.(1) is simplified to

$$\rho \frac{\partial v}{\partial t} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau \right) = 0$$  \hspace{1cm} (2)

![Figure 8: Viscosity estimated by the present UVP-base rheometry (red curve) and that measured by a torsional viscometer (blue plots) for 1wt% PAA solution from the data of impulsive spinning mode. Elasticity and yield stress are obtained by UVP simultaneously.](image)

Figure 8 shows the results of rheological properties
estimated by the present method. The shear-thinning characteristics of the viscosity in PAA solution are validly measured to have small error relative to the data measured by torsional viscometer. Simultaneously, the elasticity and the yield stress are obtained. These properties are determined only from the single-run measurement which takes only 5 seconds.

5 APPLICATION TO BUBBLY LIQUID

Rheology of bubbly liquid has already a long history beyond a century[5]. The recent topic relates to complex viscosity that provides interaction to turbulent eddies in high deformation rate with rich unsteadiness[6]. Now we are trying to detect viscoelastic property in bubbly liquid that consists of Newtonian gas-liquid combination.

Figure 9: Shape of bubbles captured by high-speed video camera. Top picture in stagnant state (thus, capillary number is zero), bottom picture for during spin at maximum capillary number of 1.2

Figure 10: Spinning responses of bubbly liquid obtained by UVP shown by angular velocity. Mixing of bubbles delays the momentum transfer in the initial stage, and provides large acceleration later for surface tension.

Figure 11 Spin responses of angular acceleration (time derivative of liquid angular velocity) for two cases with and without bubbles.

Figure 9 shows typical samples of interfacial distribution of bubbles in a part of cylindrical container. At capillary number of 1.2, many bubbles are stretched largely in circumferential direction. Figure 10 shows UVP data of single spin response to compare the effect of bubble mixing. As seen in Figure 11, which is a comparison of angular acceleration of liquid, mixing of bubbles delays the initial momentum transfer for interfacial yielding effect, and later provides large acceleration for recovery force of bubbles. This 0.2s-order time scale creates frequency characteristics of bubble rheology, which should impact the modulation of turbulence in bubbly liquid.

6 CONCLUDING REMARKS

UVP will be excellent rheometry since it obtains spatio-temporal velocity information with which closure problem in rheological models is completely solved. Spinning is one of the best configurations for capturing rheological properties because of well-posed informatics.

REFERENCES